

## CHEMICAL REACTION DIOPHANTINE EQUATIONS

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### ABSTRACT

The use of the linear diophantine equations to balance the valency number of the chemical equations is demonstrated in this study. Additionally, a few chemical equations and their balanced versions are shown.

**Keywords:** linear diophantine equations, chemical equations, Applications,

### INTRODUCTION

When atoms gain or lose electrons to yield ions, or fuse with alternative atoms to make molecules, their symbols are changed or combined to get chemical formulas that fittingly represent some species. Extending this symbolism to represent each the identities and also the relative quantities of substances undergoing a chemical (or physical) amendment involves writing and reconciliation of a chemical equation. In view of a chemist a simplest and most generally used method for balancing chemical equations is by inspection or balancing them using molecular weight. When it comes to a mathematician, it is done effectively by treating it as equations and solve using various methods. Here the chemical equations are balanced using diophantine equations. Solving Diophantine equation is closely tied to standard arithmetic and number theory. A Diophantine equation within the kind  $ax + by = c$  is a linear diophantine equations in 2 variables. If 2 comparatively prime integers  $a$  and  $b$  written during this kind with  $c = 1$ , the equation can associate infinite range of solutions. additionally there'll forever be associated infinite range of solutions if  $\gcd(a, b) = 1$ , and there are not any solutions to the equation if  $\gcd(a, b) \nmid c$ .

In this paper various chemical reactions are considered and treated as linear diophantine equation and are balanced consequently.

Method of analysis:

Consider the chemical reaction of the form,

$$p_u p_v w_r + q_u p_v q_r w_{r'} + r_u p_v q_r w_{r''} + \dots = P_u p_v q_r w_r + Q_u p_v q_r w_{r'} + R_u p_v q_r w_{r''} + \dots \quad (1)$$

(1) leads to,

$$pp' + qp'' + rp''' + \dots = PP' + QP'' + RP''' + \dots$$

$$pq' + qq'' + rq''' + \dots = PQ' + QQ'' + RQ''' + \dots \quad (2)$$

$$pr' + qr'' + rr''' + \dots = PR' + QR'' + RR''' + \dots$$

where  $u, v$  and  $w$ , were the elements within the reaction, are positive integers or zero, and  $p', q', r', \dots, P', Q', R', \dots$  indicates the unknown coefficients of the reactants and product. Equation (2) is an elementary Diophantine equation which gives all integer solutions  $[u, v, w, \dots, U, V, W, \dots]$ .

$$pu_p v_q w_r + qu_p v_q w_r + ru_p v_q w_r + \dots = Pu_p v_q w_r + Qu_p v_q w_r + Ru_p v_q w_r + \dots \quad (1)$$

(1) leads to,

$$pp' + qp'' + rp''' + \dots = PP' + QP'' + RP''' + \dots$$

$$pq' + qq'' + rq''' + \dots = PQ' + QQ'' + RQ''' + \dots \quad (2)$$

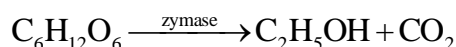
$$pr' + qr'' + rr''' + \dots = PR' + QR'' + RR''' + \dots$$

where u, v and w, were the elements in the reaction,  $p', q', r', \dots, P', Q', R', \dots$  are non-negative integers and p, q, r, ... , P, Q, R, ... are the determinable coefficients of the reactants and products.

Equation ( 2 ) is an elementary Diophantine equation which gives all integer solutions  $[u, v, w, \dots U, V, W, \dots]$ .

Several chemical equations representing the formation of various chemical products and chemical biproducts are considered and the considered equations are balanced treating them as linear diophantine equations.

1. The following is the process of preparing Ethyl Alcohol from Molasses, which has been a procedure in the Trichy Distilleries and Chemicals Ltd., Trichy.



Rewriting the above equation gives,



Here p, P, and Q are the unknowns to be identified to balance the equation.

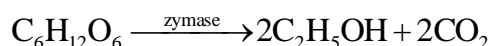
From (3), the following is obtained.

$$\left. \begin{array}{l} \text{For 'C', } p = 2P + Q \\ \text{For 'H', } 12p = 6P \\ \text{For 'O', } 6p = P + 2Q \end{array} \right\} \quad (4)$$

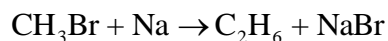
(4) gives,  $4p - 2Q = 0$ .

This is a linear diophantine equation in two unknowns which in solving gives,  $[p, Q] = [1, 2]$ .

Hence,



2. Wurtz Reaction:



Here p, q, P, and Q are the constants to be identified to balance the equation.

From ( 5 ), the following is obtained.

For 'C',  $p = 2P$

For 'H',  $3u = 6P$

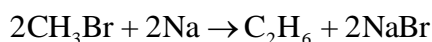
For 'Br',  $p = P$

For 'Na'  $q = Q$

which reduces to  $p = q$ .

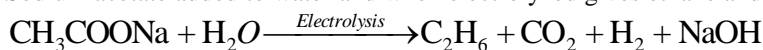
This is a linear diophantine equation in two unknowns which in solving gives,  $[p, q] = [2, 2]$ .

Hence,

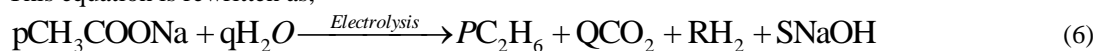


3. Kolbe's Reaction:

Sodium acetate added to water and when electrolyzed gives ethane and bi-products.



This equation is rewritten as,



Here  $p, q, P, Q, R$  and  $S$  are the constants to be identified to balance the equation.

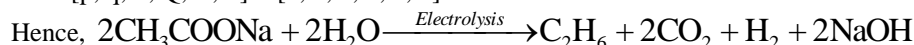
From (6), the following are obtained.

$$\left. \begin{array}{l} \text{For 'C', } p = 2P + Q \\ \text{For 'H', } 3p + 2q = 6P + 2R + S \\ \text{For 'O', } 2p + q = 2Q + S \\ \text{For 'Na', } p = S \end{array} \right\} \quad (7)$$

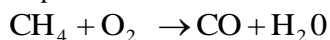
(7) reduces to,  $5p - 5q + 4R = 0$ .

This is a linear diophantine equation in three unknowns which in solving gives,  $[p, q, R] = [2, 6, 5]$ .

Thus  $[p, q, P, Q, R, S] = [2, 6, 1, 2, 5, 2]$ .



4. Preparation of water from Methane:



Now this equation is balanced treating this as a linear diophantine equation in three variables.

The considered equation is rewritten as,



where  $p, q, P, Q$  are the constants to be identified to balance the equation.

From (8), the following are obtained.

$$\left. \begin{array}{l} \text{For 'C', } p = P \\ \text{For 'H', } 4p = Q \\ \text{For 'O', } 2q = P + Q \end{array} \right\} \quad (9)$$

(9) reduces to  $3p - 2q = 0$ .

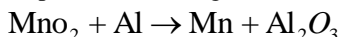
This is a linear diophantine equation in two unknowns which in solving gives,

$[p, q] = [2, 3]$ .

Thus  $[p, q, P, Q] = [2, 3, 2, 4]$ .

Hence  $2\text{CH}_4 + 3\text{O}_2 \rightarrow 2\text{CO} + 4\text{H}_2\text{O}$

5. Preparation of Manganese:



Rewriting the above equation leads to,



This on proceeding as in the previous cases reduces to,

$$2p - 3q = 0.$$

From the above equation the basic solution is obtained as,  $[p, q] = [3, 2]$ .

Thus  $[p, q, P, Q] = [3, 2, 3, 2]$ .

Hence,  $3\text{MnO}_2 + 2\text{Al} \rightarrow 3\text{Mn} + 2\text{Al}_2\text{O}_3$

S.No	Unbalanced equation	Balanced equation
1	$\text{Zn} + \text{HCl} \rightarrow \text{ZnCl}_2 + \text{H}_2$	$\text{Zn} + 2\text{HCl} \rightarrow \text{ZnCl}_2 + \text{H}_2$
2	$\text{CaO} + \text{C} \rightarrow \text{CaC}_2 + \text{CO}$	$\text{CaO} + 3\text{C} \rightarrow \text{CaC}_2 + \text{CO}$
3	$\text{N}_2 + \text{H}_2 \rightleftharpoons \text{NH}_3$	$\text{N}_2 + 3\text{H}_2 \rightleftharpoons 2\text{NH}_3$
4	$\text{H}_2 + \text{I}_2 \rightleftharpoons \text{HI}$	$\text{H}_2 + \text{I}_2 \rightleftharpoons 2\text{HI}$
5	$\text{Al} + \text{Cl}_2 \rightarrow \text{AlCl}_3$	$2\text{Al} + 3\text{Cl}_2 \rightarrow 2\text{AlCl}_3$
6	$\text{Si} + \text{F}_2 \rightarrow \text{SiF}_4$	$\text{Si} + 2\text{F}_2 \rightarrow \text{SiF}_4$
7	$\text{CaCO}_3 + \text{HCl} \rightarrow \text{CaCl}_2 + \text{H}_2\text{O} + \text{CO}_2$	$\text{CaCO}_3 + 2\text{HCl} \rightarrow \text{CaCl}_2 + \text{H}_2\text{O} + \text{CO}_2$
8	$\text{BaCl}_2 + \text{Na}_2\text{SO}_4 \rightarrow \text{BaSO}_4 + \text{NaCl}$	$\text{BaCl}_2 + \text{Na}_2\text{SO}_4 \rightarrow \text{BaSO}_4 + 2\text{NaCl}$
9	$\text{CuO} + \text{NH}_3 \rightarrow \text{Cu} + \text{N}_2 + \text{H}_2\text{O}$	$3\text{CuO} + 2\text{NH}_3 \rightarrow 3\text{Cu} + \text{N}_2 + 3\text{H}_2\text{O}$
10	$\text{HCl} + \text{O}_2 \rightarrow \text{H}_2\text{O} + \text{Cl}_2$	$4\text{HCl} + \text{O}_2 \rightarrow 2\text{H}_2\text{O} + 2\text{Cl}_2$

## CONCLUSION

There are several applications for diophantine equations. In this paper the linear diophantine equations are used to balance the valency number of chemical equations. One may also try to attempt some other procedures to do the same.

## REFERENCES

1. R.D. Carmichael, "The Theory of Numbers and Diophantine Analysis", Dover Publications, New York 1959.
2. Mordell L.J., "Diophantine Equations" Academic Press, New York, 1970.
3. Dickson. L.E. "History of Theory of Numbers and Diophantine Analysis", Vol.2, Dove Publications, New York 2005.
4. G Janaki, P Saranya, On the Ternary Quadratic Diophantine Equation  $5(x^2 + y^2) - 6xy = 4z^2$ , Imperial Journal of Interdisciplinary Research, Vol.2, Issue.3, pp.396-397, 2016.



5. G Janaki, P Saranya, On the Ternary Cubic Diophantine Equation  $5(x^2 + y^2) - 6xy + 4(x + y) + 4 = 40z^3$ , International Journal of Science and Research-Online, Vol.5, Issue.3, pp.227-229, 2016.
6. G Janaki, P Saranya, On the Quintic Non-Homogeneous Diophantine Equation  $x^4 - y^4 = 40(z^2 - w^2)p^3$ , International Journal of Engineering Science and Computing, Vol7 Issue.2, pp.4685-4687, 2017.
7. Deepinder Kaur Meenal Sambhor, Diophantine Equations and its Applications in Real Life International Journal of Mathematics And its Applications, Volume 5, Issue 2-B ,2017, pp.217–222.
8. U N Roy , R P Sah, Linear Diophantine Equation: Solution and Applications, International Journal of Mathematics Trends and Technology (IJMTT), Volume 65 Issue, January 2019, pp.09-12.
9. WA Wolovich , PJ Antsaklis, The Canonical Diophantine Equations with Applications, Siam J. Control And Optimization Vol. 22, No. 5, September 1984,pp.777-787.
10. Anbuselvi R , Nithya D, Applications of Diophantine Equations in Chemical Equations, Alochana  
a. Chakra Journal Volume IX, Issue V, May 2020 pp.1969-1973.