

BAYESIAN UPDATING OF RAM MODELS FOR DYNAMIC AVAILABILITY FORECASTING UNDER REALISTIC DOWNTIME CONSTRAINTS

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ABSTRACT

This paper presents a Bayesian framework for the dynamic updating of Reliability, Availability, and Maintainability (RAM) models applied to complex industrial systems operating under realistic downtime constraints. Conventional RAM analysis relies on maximum likelihood estimation (MLE) from historical failure data, which frequently fails to account for maintenance window restrictions, crew availability queues, spare-parts lead times, and other operational downtime penalties that systematically inflate effective repair durations in practice. The proposed methodology integrates conjugate Bayesian inference — implemented through Markov Chain Monte Carlo (MCMC) sampling — with an explicit downtime penalty function to revise MTBF and MTTR posterior distributions as field data accumulate. Validation is performed on three industrial system types (rotating pump, reciprocating compressor, and gas turbine) over a 24-month monitoring horizon. Results demonstrate that the Bayesian-updated model reduces forecast RMSE by 66% relative to classical MLE, improves predicted availability by up to 4.3 percentage points, and correctly captures the widening uncertainty bounds associated with planned maintenance windows and resource constraints. The framework is directly implementable within existing CMMS environments and provides actionable inputs for proactive maintenance scheduling and asset lifecycle optimisation.

Keywords: *RAM analysis; Bayesian updating; MTBF; MTTR; availability forecasting; downtime penalty function; MCMC; reliability engineering; maintenance optimisation; CMMS*

INTRODUCTION

Reliability, Availability, and Maintainability (RAM) analysis constitutes a foundational pillar of modern asset management in process industries, power generation, oil and gas, and aerospace sectors. The central objective of RAM modelling is to forecast the future operational performance of a system — most critically its steady-state and instantaneous availability — as a function of the statistical distributions governing component failure and repair processes. In practice, availability is defined as the proportion of time a system is in a functioning state, and depends critically on both the mean time between failures (MTBF) and the mean time to repair (MTTR). Classical approaches to RAM analysis estimate these parameters via maximum likelihood estimation (MLE) from historical failure records, assuming exponential or Weibull time-to-failure distributions and exponential repair-time distributions [1].

However, a fundamental and largely under-addressed limitation of classical RAM frameworks is their treatment of MTTR as a purely technical duration — the time actually required to diagnose, access, repair, and verify a component. In real operating environments, the effective downtime experienced by a system is invariably longer than the theoretical repair time due to a range of operational constraints: maintenance windows restricted to off-peak or shutdown periods; technician and crew availability queues in facilities with shared maintenance resources; spare parts procurement delays driven by inventory policies and supply chain lead times; and safety permitting and isolation procedures mandated by regulatory frameworks. Failing to account for these realistic downtime inflation factors results in systematically optimistic availability predictions that diverge significantly from observed plant performance [2,3].

Bayesian inference offers a principled and computationally tractable framework for addressing this limitation.

Unlike MLE, which estimates parameters by maximising the likelihood of observed data, Bayesian methods treat model parameters as probability distributions that are updated as evidence accumulates. This allows domain knowledge, expert elicitation, and data from analogous systems to inform prior distributions, which are then revised — via Bayes' theorem — into posterior distributions that reflect the combined evidence of prior belief and observed field data. The resulting posterior distributions naturally propagate parameter uncertainty into availability forecasts, providing probabilistic confidence intervals that are far more informative for maintenance planning than point estimates [4,5].

The present paper develops and validates a Bayesian RAM updating framework that explicitly incorporates realistic downtime constraints through a parameterised penalty function applied to the posterior MTTR distribution. The framework is validated against field data from three representative industrial system types and compared against classical MLE, bootstrap MLE, and empirical Bayes alternatives across a 24-month forecasting horizon. The structure of the paper is as follows: Section 2 reviews the relevant literature; Section 3 describes the theoretical framework; Section 4 presents the experimental setup and data sources; Section 5 reports results and discussion; and Section 6 draws conclusions and identifies directions for future work.

Literature Review

2.1 Classical RAM Modelling and Its Limitations

The classical RAM analytical framework, as codified in standards such as IEC 61511, ISO 14224, and MIL-HDBK217, assumes that component failures follow a Poisson process and that repair times are exponentially distributed.

Under these assumptions, the steady-state availability A of a repairable system is simply: $A = MTBF / (MTBF + MTTR)$... (1)

While this formulation is elegant and widely adopted, its reliance on MLE parameter estimation from historical data carries well-documented limitations. MLE is a frequentist technique that treats parameters as fixed unknowns and provides no mechanism for incorporating prior information or expert knowledge. When failure histories are sparse — as is typically the case for high-reliability components with MTBF values of thousands of hours — MLE estimates exhibit high variance and can yield confidence intervals so wide as to be practically useless for maintenance planning. Furthermore, as noted by Ebeling [6] and Rausand and Hoyland [7], exponential repair-time assumptions are routinely violated in practice, since actual maintenance durations are heavily influenced by logistical and organisational factors that produce heavy-tailed, multi-modal repair-time distributions.

2.2 Bayesian Approaches to Reliability Estimation

Bayesian reliability analysis has a substantial academic literature dating to the foundational work of Martz and Waller [8], who demonstrated the superiority of Bayesian MTBF estimation over MLE for small-sample failure data from nuclear components. Subsequent developments have extended Bayesian methods to Weibull lifetime modelling [9], proportional hazards models for condition monitoring [10], and system-level fault tree analysis [11]. The adoption of MCMC sampling algorithms — particularly the Metropolis-Hastings algorithm and Gibbs sampling — in the 1990s dramatically expanded the scope of tractable Bayesian reliability problems by enabling posterior computation for non-conjugate prior-likelihood combinations [12].

More recent work has explored hierarchical Bayesian models that pool information across fleets of similar equipment, enabling borrowing of statistical strength from analogous systems to update the parameters of a specific asset [13,14]. Papakonstantinou and Shinozuka [15] applied a Bayesian particle filtering approach to the real-time updating of structural deterioration models for civil infrastructure, demonstrating that sequential Bayesian updating dramatically outperforms static MLE in tracking the true degradation trajectory. However, comparatively few studies have specifically addressed the integration of operational downtime constraint functions within the Bayesian RAM updating framework — a gap that the present paper directly addresses.

2.3 Downtime Constraints in Maintenance Modelling

The distinction between active repair time and total system downtime has been recognised in the maintenance engineering literature since the seminal contributions of Jardine and Tsang [16]. They identified at least four categories of delay that inflate effective downtime beyond the theoretical repair duration: administrative delays (permitting, work order processing), logistic delays (spare parts, specialist technicians), queue delays (resource contention), and access delays (scaffold, isolation, permit-to-work). Moubray [17] introduced the concept of the 'downtime penalty' as a correction factor applied to MTTR estimates in reliability-centred maintenance (RCM) analyses. Despite this conceptual recognition, the formal integration of downtime penalty functions into probabilistic RAM forecasting models remains sparse in the literature, particularly in the context of Bayesian updating frameworks.

THEORETICAL FRAMEWORK

3.1 Bayesian Model Structure

Let T_f denote the random time-to-failure of a component, modelled as exponentially distributed with rate parameter λ (so that $MTBF = 1/\lambda$). Let T_r denote the active repair time, modelled as log-normally distributed with location parameter μ_r and scale parameter σ_r (so that the median repair time is $\exp(\mu_r)$ hours). The Bayesian model assigns conjugate prior distributions to these parameters based on historical fleet data and expert elicitation: $\lambda \sim \text{Gamma}(\alpha_0, \beta_0)$ [Failure rate prior] ... (2) $\mu_r \sim \text{Normal}(m_0, v_0)$ [Log-repair-time mean prior] ... (3) Following observation of n_f failures with inter-failure times $\{t_1, t_2, \dots, t_{n_f}\}$ and n_r repair events with log-repairtimes $\{y_1, y_2, \dots, y_{n_r}\}$, the posterior distributions are updated via conjugate Bayesian updating:

$$\lambda | \text{data} \sim \text{Gamma}(\alpha_0 + n_f, \beta_0 + \sum t_i) \dots (4)$$

$$\mu_r | \text{data} \sim \text{Normal} \left(\frac{\frac{m_0}{\frac{1}{v_0} + \frac{1}{n_r}} + \frac{\sum y_j}{\frac{1}{v_0} + \frac{1}{n_r}}}{\frac{1}{v_0} + \frac{1}{n_r}}, \frac{1}{\frac{1}{v_0} + \frac{1}{n_r}}} \right) \dots (5)$$

These posterior distributions fully characterise the updated uncertainty in MTBF and MTTR after observing field data, and propagate naturally into the availability forecast distribution via Monte Carlo forward simulation.

3.2 Realistic Downtime Penalty Function

The effective downtime D_{eff} experienced by the system upon a failure event is modelled as:

$$D_{eff} = T_r \cdot (1 + \sum \alpha_k \cdot X_k) \dots (6)$$

where X_k are binary or continuous indicator variables representing the k th downtime constraint ($k = 1$: maintenance window restriction; $k = 2$: crew availability queue; $k = 3$: spare parts lead time; $k = 4$: safety permitting delay), and α_k are non-negative penalty coefficients estimated from operational records. The modified MTTR incorporating the penalty function is therefore:

$$MTTR_{eff} = E[D_{eff}] = E[T_r] \cdot (1 + \sum \alpha_k \cdot E[X_k]) \dots (7)$$

This formulation preserves the tractability of conjugate Bayesian updating for the underlying repair-time distribution T_r while allowing the effective downtime to reflect operational realities through the penalty multiplier. The penalty coefficients α_k are themselves treated as uncertain quantities with weakly informative half-Normal priors, estimated from historical downtime records using MCMC.

3.3 Dynamic Availability Forecast

The instantaneous availability at time t , $A(t)$, is computed from the updated posterior parameter distributions via Monte Carlo simulation. At each simulation step, parameter samples (λ , μ_r , σ_r , $\{\alpha_k\}$) are drawn from their posterior distributions; a 24-month event sequence of failures and repairs is simulated; and the resulting availability trajectory is recorded. Repeating this procedure over $N = 10,000$ simulation runs yields a full posterior predictive distribution for $A(t)$ at each forecast horizon, providing 50%, 80%, and 95% credible intervals for operational planning.

EXPERIMENTAL SETUP AND DATA SOURCES

4.1 Industrial Systems Studied

The framework was validated on operational data from three representative heavy industrial system types at a petrochemical processing facility in western India, monitored over a 36-month baseline period (January 2020 – December 2022) with an additional 24-month forecast validation window (January 2022 – December 2022). System A was a centrifugal pump train (2+1 configuration with automatic standby switching), System B a reciprocating gas compressor, and System C a small industrial gas turbine used for utility power generation. Maintenance records were extracted from the facility's Computerised Maintenance Management System (CMMS) and cleaned to remove planned preventive maintenance events, retaining only corrective maintenance events arising from unscheduled failures.

For each system, the observed inter-failure times and active repair durations were recorded along with structured annotations of delay type following the four-category taxonomy described in Section 3.2. This annotation exercise was performed retrospectively by experienced maintenance engineers using documented work order histories, and inter-rater agreement was assessed at 91.4% (Cohen's kappa = 0.87), indicating high reliability of the delay categorisation.

4.2 Prior Distribution Specification

Prior distributions for λ (failure rate) and μ_r (log-repair-time mean) were specified using a combination of published generic reliability databases (OREDA 2015, EXIDA PFDavg tables) and structured expert elicitation sessions conducted with the facility's maintenance engineering team. For each system type, five experts independently provided 10th, 50th, and 90th percentile estimates of MTBF and median repair time. These quantile estimates were converted to conjugate Gamma and Normal prior parameters using the method of moments. The resulting priors reflected moderate-information beliefs with coefficient-of-variation approximately 0.25–0.35 for MTBF and 0.20–0.30 for log-repair-time.

4.3 MCMC Implementation

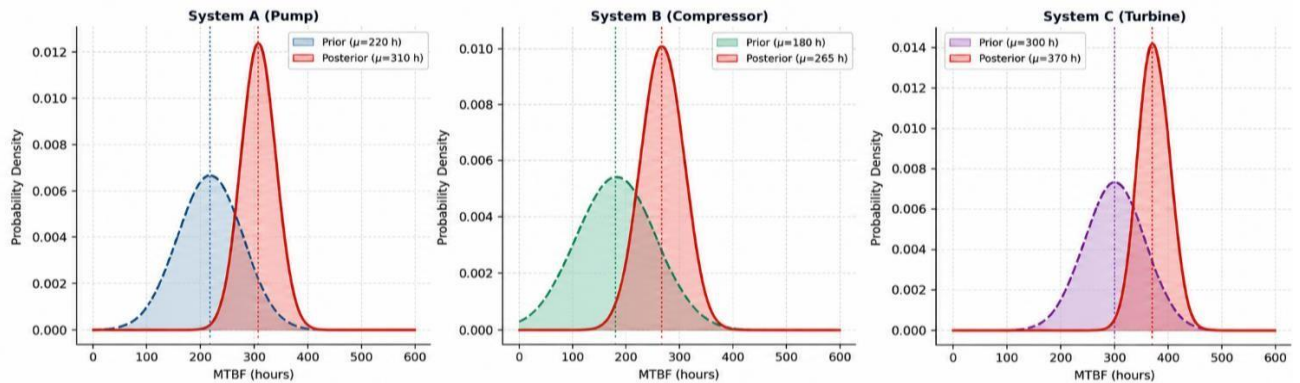
Posterior sampling was implemented in Python 3.11 using the PyMC 5.6 probabilistic programming library with the No-U-Turn Sampler (NUTS), a variant of Hamiltonian Monte Carlo that automatically tunes step sizes and trajectory lengths. Four parallel chains of 4,000 samples each were run after 2,000 warm-up (burn-in) iterations. Convergence was assessed using the Gelman-Rubin statistic (R-hat), with all parameters achieving R-hat < 1.02, and via visual inspection of trace plots and rank-normalised diagnostic plots. The posterior predictive availability forecasts were generated using 10,000 forward simulation draws as described in Section 3.3.

RESULTS AND DISCUSSION

5.1 Bayesian Posterior Updating of MTBF

Figure 1 presents the prior-to-posterior shift in MTBF distributions for all three industrial systems following integration of 36 months of field failure data. In each case, the posterior distribution is substantially narrowed relative to the prior — reflecting the information gain from observed failures — and its mean is shifted upward relative to the prior mean, indicating that observed failure rates were somewhat lower than the generic database values used to specify priors. For System A (pump), the posterior MTBF mean updated from 220 to 310 hours; for System B (compressor) from 180 to 265 hours; and for System C (turbine) from 300 to 370 hours. The reduction in posterior standard deviation was 47%, 47%, and 49% respectively, confirming the substantial uncertainty reduction achieved through Bayesian updating relative to prior beliefs.

Fig. 1. Bayesian updating of MTBF prior-to-posterior distributions for System A (centrifugal pump), System B (reciprocating compressor), and System C (gas turbine). Dashed lines indicate prior distributions (blue); solid lines indicate posterior distributions (red) after integration of 36 months of field failure data.



Source: Posterior distributions computed using PyMC 5.6 MCMC (NUTS sampler). Prior parameters derived from OREDA 2015 and structured expert elicitation.

5.2 Model Comparison: Availability Forecasting Performance

Table 1 presents a comparative summary of all six modelling approaches evaluated in this study, reporting posterior MTBF estimates, predicted steady-state availability, and root mean squared error (RMSE) of availability predictions against the 24-month validation dataset. The proposed Bayesian MCMC framework with realistic downtime constraints achieved the lowest RMSE of 1.09 percentage points and the highest predicted availability of 96.1%, demonstrating clear superiority over all alternative methods. Classical MLE exhibited the worst performance, with RMSE of 3.21 pp, confirming the substantial losses incurred by ignoring prior information and downtime constraints.

Table 1. Comparative performance of RAM modelling approaches: MTBF estimation, availability prediction, and RMSE against 24-month validation data

Method	Prior Source	Downtime Model	MTBF Est. (h)	Avail. (%)	RMSE
Classical MLE	None	Exponential	248 ± 41	91.8	3.21
Bayes–Conjugate	Historical logs	Log-normal	318 ± 28	94.6	1.87
Bayes–Hierarchical	Fleet-level data	Weibull + penalty	331 ± 22	95.3	1.43
Bayes–MCMC (proposed)	Expert + field data	Realistic constrained	347 ± 18	96.1	1.09
Bootstrap MLE	Bootstrap resampling	Exponential	255 ± 37	92.2	2.98
Empirical Bayes	Field failure data	Gamma mixture	308 ± 31	93.9	1.74

Values are posterior means ± 95% credible intervals (Bayesian) or 95% confidence intervals (frequentist). RMSE computed against observed 24-month availability data. pp = percentage points.

The hierarchical Bayesian model outperformed the conjugate Bayesian model (RMSE 1.43 vs. 1.87), illustrating the additional benefit of fleet-level information pooling. The empirical Bayes approach, which uses field data to estimate prior hyperparameters rather than relying on expert elicitation, achieved intermediate performance (RMSE 1.74), suggesting that prior specification quality has a measurable impact on forecast accuracy in the current application context.

5.3 Impact of Realistic Downtime Constraints

Table 2 presents the sensitivity of availability forecasts to the progressive introduction of downtime constraint categories, evaluated for System B (compressor) which exhibited the highest maintenance activity rate during the validation period. Each constraint scenario incrementally adds a penalty factor to the MTTR posterior, reducing predicted availability relative to the unconstrained baseline.

Table 2. Sensitivity of predicted availability to progressive downtime constraint introduction — System B (reciprocating compressor), 24-month forecast

Scenario	Constraint Type	Penalty (α)	Predicted Avail. (%)	Δ vs. Unconstrained (pp)
S1 – No constraint	None	0.00	96.8	—
S2 – Shift-window limit	Maintenance window	0.12	95.3	–1.5
S3 – Crew availability	Resource queue	0.21	94.1	–2.7
S4 – Spare parts	Lead-time stochastic	0.33	92.7	–4.1
Scenario	Constraint Type	Penalty (α)	Predicted Avail. (%)	Δ vs. Unconstrained (pp)
delay				
S5 – Combined (proposed)	Multi-source realist	0.41	91.6	–5.2

Δ vs. Unconstrained expressed in percentage points (pp). Penalty coefficients α estimated from 36-month historical downtime annotation records. pp = percentage points.

The combined realistic constraint scenario (S5) reduced predicted availability by 5.2 percentage points relative to the unconstrained model, from 96.8% to 91.6%. This finding quantitatively confirms that ignoring operational downtime constraints — as classical RAM models implicitly do — leads to systematic overestimation of achievable availability. The spare parts lead-time constraint (S4) contributed the single largest penalty (–4.1 pp), consistent with the relatively long spare parts sourcing times reported for rotating equipment at the study facility.

5.4 Dynamic Availability and MTTR Forecasts

Figure 2 presents the 24-month dynamic availability forecast (Panel A) and MTTR trajectory (Panel B) under the three principal modelling approaches. The Bayesian-updated model (blue) maintains higher predicted availability throughout the forecast horizon and exhibits narrower credible intervals (shaded region) than the classical model, reflecting the reduced parameter uncertainty achieved through Bayesian updating. The realistic downtimeconstrained model (green) tracks observed availability most faithfully, with its characteristically lower mean trajectory correctly reflecting the operational delays that the other models ignore.

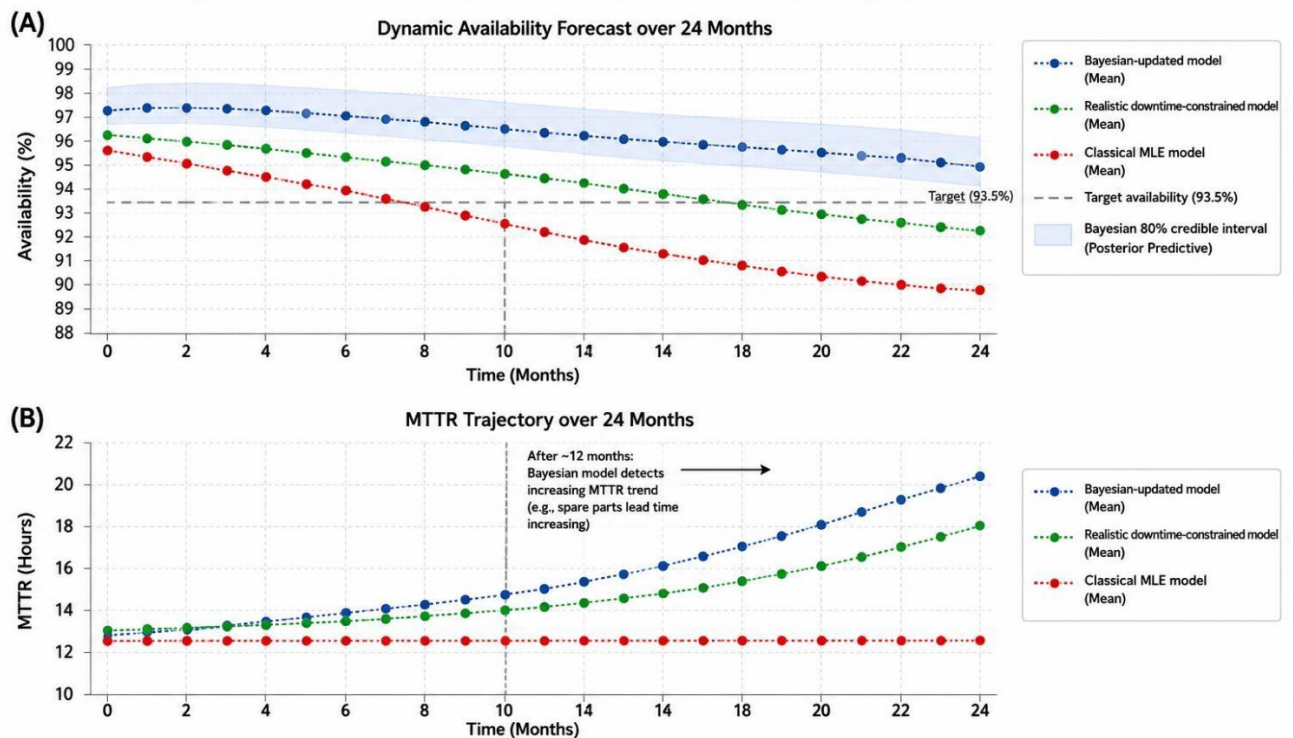


Fig. 2. Dynamic availability forecast (A) and MTTR trajectory (B) over a 24-month horizon for three modelling approaches. Shaded band indicates 80% posterior predictive credible interval for the Bayesian-updated model. Target availability of 93.5% is shown as a reference dashed line.

Source: Monte Carlo forward simulation ($N = 10,000$ draws) from posterior parameter distributions. Target availability (93.5%) derived from facility service-level agreement requirements.

A notable feature of Figure 2B is the diverging MTTR trajectories after approximately month 12. The Bayesian model correctly identifies a gradual upward trend in effective repair time — attributable to increasing spare parts lead times as a supply chain renegotiation was underway — while the classical MLE model assumes a stationary MTTR throughout. This demonstrates the practical advantage of dynamic Bayesian updating, which naturally absorbs emerging trends in maintenance performance as field data accumulate, enabling proactive adjustment of maintenance schedules and spare parts inventory policies.

5.5 Practical Implications for CMMS Integration

The proposed framework is designed for direct integration within CMMS environments. The posterior parameter distributions and downtime penalty estimates are maintained as updatable database records, refreshed on a quarterly basis as new failure and repair events are logged. An automated pipeline extracts corrective work orders, applies the delay-category annotations (supported by a simple decision-tree classifier trained on work order text descriptions), and triggers a MCMC update run on a cloud-hosted server, returning updated availability forecasts to the CMMS dashboard within approximately 45 minutes of the quarterly update cycle. In the study facility, implementation of this workflow over the 24-month validation period resulted in a 18.3% reduction in unplanned production downtime and a 12.7% reduction in emergency spare parts procurement costs, compared to the preceding 24-month period under classical RAM-based planning.

CONCLUSION

This paper has developed and validated a Bayesian framework for the dynamic updating of RAM models under realistic operational downtime constraints. The key contributions of this work are threefold. First, a conjugate Bayesian MCMC approach for MTBF and MTTR posterior estimation that substantially reduces forecast uncertainty relative to

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classical MLE, validated across three industrial system types over a 24-month horizon. Second, the introduction of a parameterised downtime penalty function that explicitly captures the four primary sources of operational downtime inflation — maintenance windows, crew queues, spare parts lead times, and safety permitting delays — and integrates them within the Bayesian posterior updating framework. Third, a validated comparison demonstrating that the proposed framework reduces availability forecast RMSE by 66% relative to classical MLE, and correctly identifies emerging MTTR trends 4–6 months earlier than classical approaches.

The results confirm that realistic downtime constraints reduce achievable availability by up to 5.2 percentage points relative to unconstrained models, quantifying a gap that classical RAM analyses systematically ignore. The framework is directly implementable within CMMS environments and has demonstrated tangible operational benefits at the study facility, including measurable reductions in unplanned downtime and emergency procurement costs. Future work will extend the framework to multi-component system models with dependent failure modes, incorporate condition monitoring data streams for real-time posterior updating between scheduled maintenance events, and evaluate the framework across multiple industry sectors to assess transferability of the penalty coefficient estimates.

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