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INFLUENCE OF SHORTENED NOZZLE SHAPE WITH A BELL-SHAPED TIP ON THE GAS FLOW CHARACTERISTICS

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***ABSTRACT*:**

Interest in nozzles with great shortened profiles arose in connection with the development of high-density aircraft. A shortened nozzle profile significantly reduces or eliminates the free volumes required to accommodate a full-size nozzle contoured along the flowline. This paper examines flow in a similar nozzle with various possible combinations of a shortened Laval nozzle section with a bell-shaped tip with an exit angle of 0°. The aim is to investigate the gas flow characteristics in a shortened rocket engine nozzle with a bell-shaped tip under varying nozzle geometry and pressures upstream of the nozzle and the ambient air. The study found that the relative length of the shortened conical section of the nozzle, located upstream of the nozzle, has a significant effect on the flow characteristics in the nozzle. The overall nozzle length (with the same geometric expansion ratio) and the nozzle inlet angle have less influence on the flow pattern in the nozzle. At low nozzle inlet pressures and high external pressure, a well-developed separation zone is observed between the jet boundary and the nozzle wall, extending upstream from the tip exit to the exit of the shortened nozzle. The characteristics of this zone depend on the flow expansion ratio in the shortened nozzle. The nozzle thrust coefficient decreases with increasing relative pressure at the nozzle inlet due to the reduced influence of external pressure on the flow in the tip. At higher flow misalignments, the thrust coefficient is higher for a nozzle with a longer conical section.

Keywords: *Bell-Shaped Tip, Nozzle Characteristics, Pressure, Shortened Nozzle, Thrust Coefficient*

INTRODUCTION

Modern rocket propulsion systems use various nozzle types: Laval, bell-shaped, double-bell, annular and others. The fundamental characteristics of the Laval nozzle have been well studied, and various design variations have been proposed [Cardenas, et al., (2025); Mohan Kumar G1, et al., (2013)]. However, certain nozzle operating conditions can cause over- or under-expansion of the flow, reducing its efficiency. Therefore, researchers are pursuing various modifications to the standard nozzle. Research is underway on the development of shortened nozzles based on the "classic" Laval nozzle. A minimum-length nozzle was designed in [Anuj Kumar, et al. (2020)] using the method of characteristics. This method has been very popular for nozzle contour design. For example, the authors of [Yunus Emre Özkan (2021)] used the method of characteristics to design a supersonic nozzle for two-dimensional, steady, inviscid, isentropic, and irrotational flow.

For new problems, researchers have turned to shortened nozzles with reduced length and mass. The authors [Murrough Murnaghan (2019)] used classic conical nozzles or those profiled using the characteristic method as a base and then shortened them. This caused minor jumps in the nozzle and slightly increased wave losses of nozzle thrust. The nozzle shape also changed. Recently, a bell-shaped nozzle has been considered in rocket engines, which has a larger entry angle into the supersonic nozzle part compared to the Laval nozzle [Prapti Joshi, et al., (2020)]. For solving certain problems of rocket technology this type of nozzle has advantages over the Laval nozzle in terms of size and efficiency, and reduces the complexity of manufacture and use compared, for example, with annular nozzles with a central body [Cardenas, et al., (2025)]. The processes occurring in a bell-shaped nozzle are similar to the processes in a Laval nozzle. In [Strelnikov, H.O., et al., (2025): Asha, et al (2021)], different types of nozzle outlet configurations are considered: conical and profiled. Conical nozzles are often used due to the ease of manufacture. However, it is preferable to use a profiled outlet, since it increases the nozzle efficiency while reducing its mass. The bell-shaped design provides a large expansion angle just after the throat. Then it decreases to provide a small angle at the nozzle exit. Expansion waves are generated just after the throat [Cardenas, et al., (2025)]. Farther downstream, compression shock waves are generated in the bell-shaped nozzle due to the large gradient of the wall slope change. As a rule, complex vortex flows are generated. Double-bell nozzles are currently being considered [Sreenath K.R, (2016)]. The double-bell nozzle has better overall efficiency with changing flight altitude than the single-bell nozzle. Air pressure limits the expansion of the exhaust gases at low altitudes, so the efficiency at low altitudes is much higher. It can also expand the engine exhaust to a larger effective nozzle area at high altitudes. In [Sreenath K.R, (2016)], four different types of Bell and double-bell nozzles were selected and investigated. It was shown that the correct choice of the contour geometry makes it possible to increase the design efficiency at different altitudes. The double-bell nozzle is altitude-adaptable and has two operating modes. The study [Cardenas, I.R. (2025); Chloé Génin, (2021)] identified the influence of nozzle geometry on flow behaviour and, consequently, on nozzle efficiency. Using the characteristic method, it was not possible to study the complex flow properties in a bell-shaped nozzle. Recently, research [Cardenas, I.R. (2025); Sreenath K.R, (2016); Balaji Krushna (2013)] has been conducted using computational fluid dynamics (CFD) that has enabled the discovery of new patterns in nozzle flow behaviour.

Interest in nozzles with strong shortened profiles arose in connection with the development of high-density aircraft. The attention of high-density aircraft developers has been drawn to new circular nozzle configurations that allow for a significant reduction or elimination of the free volumes required to accommodate a full-length (given expansion ratio) streamlined nozzle. Attempts to compact the layout while maintaining the nozzle length constant (for example, by using a recessed nozzle, sliding sectional nozzles, etc.) usually lead to unreasonably greater complications in the nozzle block design and a decrease in its energy-mass efficiency within the aircraft. This also creates additional difficulties in solving issues of engine thrust vector control and the placement of its auxiliary units. For solid-fuel multistage rockets, it was energetically justified to compress not the entire contour, but only the nozzle portion necessary to reduce the longitudinal dimensions of the rocket and accommodate auxiliary elements (in particular, the engine bottom of the previous rocket stage and its thrust vector control). This also allows protecting the gas-dynamic thrust vector controls from the condensed phase of the product flow.

In this regard, the optimal solution is the simple design use of a shortened nozzle with gas-dynamic control of the thrust vector (in particular, the disturbance of the supersonic flow by an obstacle on the nozzle wall). In this case, the configuration of the shortened nozzle section, as a rule, has large gradients of the contour generatrix, determined by the conditions of the aircraft layout. It should be noted that it is necessary to solve a complex design problem [Strelnikov, H.O., et al., (2025)] when you choose the optimal nozzle type, its design scheme, the geometry of the flow section, the methods and devices for controlling the thrust vector in magnitude and direction. The solution of this problem is associated, on one hand, with complex gas-dynamic calculations of viscous and inviscid high-temperature multiphase gas flow in the nozzle, taking into account its interaction with the external environment. On the other hand, this should be accompanied by complex experiments, research of energy-mass characteristics and comparison of many variants of designs and parameters of nozzle blocks, taking into account the conditions of their application. The use of the method of characteristics did not allow the investigation of the complex flow features in a bell-shaped nozzle. Recently, research has been conducted using computational fluid dynamics (CFD) [Cardenas, I.R. (2025); Sreenath K.R, (2016); Balaji Krushna (2013)], that has enabled the discovery of new patterns of nozzle flow behaviour.

In connection with the modification and optimization of the shortened nozzle contour, it is important to understand the effect of reducing the nozzle length with tip on the flow pattern within it. This paper examines the flow in such a nozzle with various possible combinations of a shortened Laval nozzle section with a bell-shaped nozzle with an exit angle of 0°.

The purpose of paper is to investigate the gas flow characteristics in a shortened rocket engine nozzle with a bell-shaped tip under varying nozzle geometry, upstream pressure, and ambient pressure.

MATERIALS AND METHODS

The study investigates shortened nozzles based on the Laval nozzle with bell-shaped tips. Fig. 1 shows the geometric parameters of the basic nozzle and a photograph of its model.

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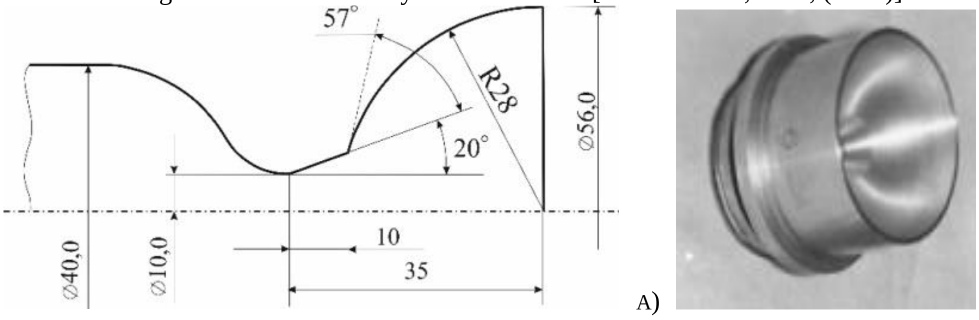
The calculations were performed on a section measuring 30L x 15L, where L is the nozzle length. The solution was found based on the Reynolds-averaged Navier–Stokes equations in an unsteady axisymmetric formulation using the kω SST turbulence model with wall functions and a compressibility correction. This turbulence model was selected taking into account the analysis conducted in [G. Strelnikov, et. al, (2022)].



a) – model sketch; b) – model photograph

Fig. 1 Model of a shortened Laval nozzle with a bell-shaped tip

The calculations were performed on a section measuring 30L x 15L, where L is the nozzle length. The solution was found based on the Reynolds-averaged Navier–Stokes equations in an unsteady axisymmetric formulation using the kω SST turbulence model with wall functions and a compressibility correction. This turbulence model was selected taking into account the analysis conducted in [G. Strelnikov, et. al, (2022)]. Air with an adiabatic index of $\gamma = 1.4$ is used as the operating medium. The calculations were performed at nozzle inlet pressures of P_0 = 50, 100, 200, and 300 bar. The ambient pressure was taken P_H = 1 bar, which corresponded to the position on the earth's surface, and P_H = 0.1 and 0.01 bar for flights in the upper atmosphere.

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RESULTS AND DISCUSSIONS

3.1 Changing the Conical Section Length

Let's examine the effect of changing the conical section length of a shortened nozzle with a bell-shaped tip on the flow wave structure in and behind the nozzle. Using the base nozzle (Fig. 1), three nozzles with conical sections of different lengths were studied (Fig. 2).

Fig. 3 shows the flow velocity distributions at P_0 = 100 bar, P_H = 1 bar in shortened Laval nozzles with different conical section lengths. The conical nozzle section generates a supersonic flow, outlet velocity of which is determined by the geometric expansion ratio of the shortened nozzle section. The nozzles in Fig. 3 have different conical section lengths. The flow in the nozzle with a short conical section has higher potential energy at the outlet than in nozzle with a long one. Moreover, the static pressure at the exit of the long section is lower than the static pressure at the exit of short one.

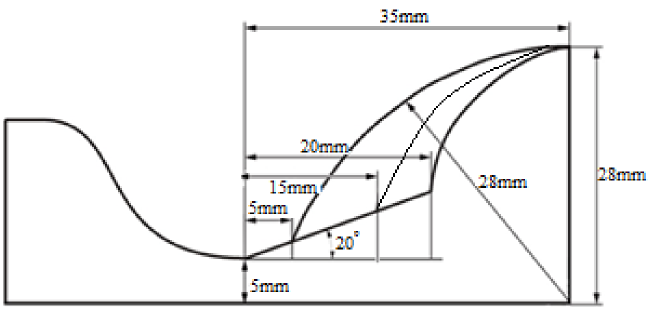

Fig. 2 Geometric parameters of shortened nozzles

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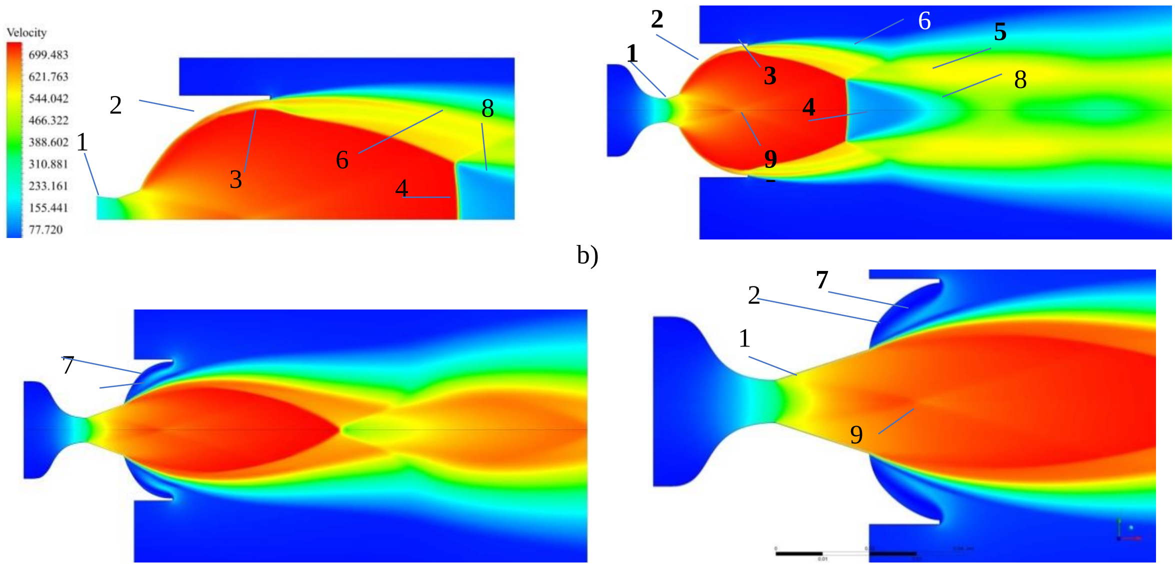

b)

c) d)

1 – shortened Laval nozzle; 2 – bell-shaped tip; 3 – hanging shock; 4 – Mach disk; 5 – reflected shock; 6 – secondary hanging shock; 7 – separation zone; 8 – subsonic zone behind the Mach disk; 9 – saddle-shaped shock wave structure

Fig. 3 Flow velocity distribution in nozzles with a conical section length: a) *l* = 5 mm; b) *l* = 10 mm; c) *l* = 15 mm; d) *l* = 20 mm

The conical nozzle section generates a supersonic flow, outlet velocity of which is determined by the geometric expansion ratio of the shortened nozzle section. The nozzles in Fig. 3 have different conical section lengths. The flow in the nozzle with a short conical section has higher potential energy at the outlet than in nozzle with a long one. Moreover, the static pressure at the exit of the long section is lower than the static pressure at the exit of short one.

Let's examine in more detail the wave structure of the flow in the nozzles shown in Fig. 3. For conical section lengths l = 5 (fig. 3 a) and 10 mm (Fig. 3 b), a Mach disk exists in the flow, while for l = 15 and 20 mm (Fig. 3 c) and 3 d), respectively), the Mach disk changes into a Mach structure. Subsonic flow follows the Mach disk, and for the last two nozzles (Fig. 3c, 3d), flow deceleration occurs in a system of oblique shocks.

Nozzles with a short conical section don't have a developed separation vortex zone near the nozzle wall, as the atmosphere does not enter the nozzle. At the nozzle wall, from the entry corner to the exit, a pre-separation zone behind the hanging shock wave is observed. Its appearance in front of the nozzle wall is explained by the expanded supersonic flow onto the wall. If the conical section length (1) increases slightly (l = 10 mm), the wave flow structure has a similar character (Fig. 3 b): a pre-separation zone of flow deceleration is observed at the nozzle wall (2) behind the detached shock wave (3), the Mach disk (4), and behind it, a subsonic zone (8). For these nozzles, well-defined wave structures are observed within the first "barrel" – the so-called "saddle-shaped" compression wave (9).

In the next two nozzles (Fig. 3 c – d) with longer conical sections, the wave structure changes. First of all, the flow separation zone (7) at the nozzle wall (2) behind the detached shock wave (3) increases. The flow structure up to the first Mach disk changes, and the shape and dimensions of this flow part depend on the nozzle length. As the conical nozzle section increases, the nozzle compression occurs (Fig. 2). This causes a decrease in the Mach disk and an extension of the first flow barrel (Fig. 3 d). The separation zone behind the corner nozzle point increases. This is due to an increase in the kinetic flow energy with an increase in the cone length and compression of the tip, and, consequently, a change in the flow structure in the near-wall region, which leads to a change in the entire flow pattern. Thus, the transverse dimensions of the near-wall vortex structure (9) increase. The indicated wave and vortex structures depend on the inlet and outer pressure value, i.e. on the engine mode and flight conditions. In this case (Fig. 3), engine operation is considered at sea level under terrestrial conditions at P_H = 1 bar. Then, an overexpanded flow from the conical section is observed, which has even more specific character at higher values of the initial pressure (P_0 = 200 - 300 bar).

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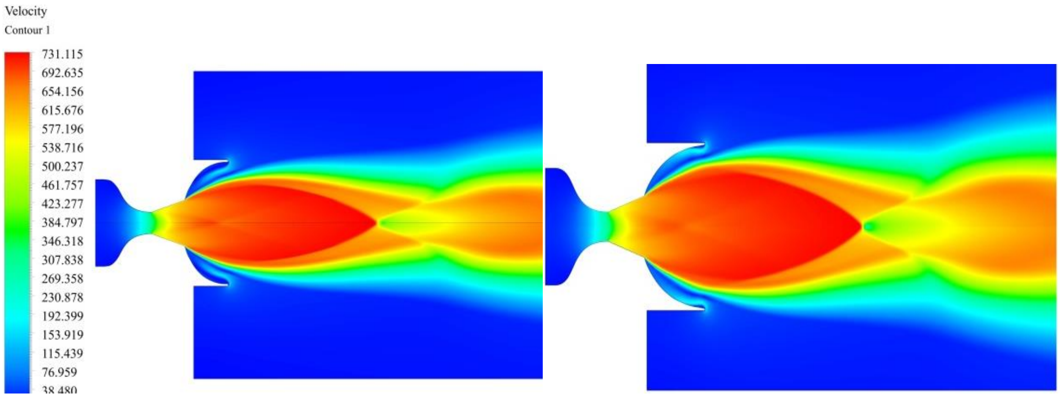
3.2. Changing the overall nozzle length

Nozzles with the same conical section length (l = 15 mm) and different bell-shaped nozzle lengths are considered, with the overall nozzle length varying: L = 35 mm, 40 mm. Environmental conditions are maintained as specified above: P_0 = 100 bar, P_H = 1 bar. Fig. 4 shows the flow velocity distribution in nozzles with different overall lengths.



a) b)

Fig. 4 Flow velocity distribution in nozzles with different overall lengths: L = 35 mm (a), L = 40 mm (b)

Research has shown that the vortex structure behind the nozzle corner changes with changes in the overall nozzle length and tip wall radius. For a longer nozzle, with increasing inlet pressure (P_0 greater than 100 bar), the flow adheres to the tip wall, which is explained by flow underexpansion at higher initial pressure. The length of the first "barrel" remains hardly changed with increasing nozzle length.

3.3. Influence of the tip entry angle on the shortened nozzle characteristics

The studied shortened nozzles have the same supersonic conical parts and the same overall nozzle length but different angles of mating (connecting) of the nozzle conical part and the bell-shaped tip: 30° (angle between the tangent to the wall and the nozzle axis is θ =50°), 50° and 70°. Such nozzle forms can serve as alternative options for shortened nozzles when designing engines. In Fig. 5 the geometry of the observed nozzles is shown

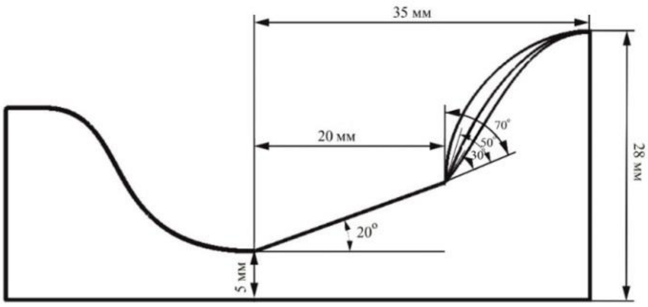


Fig. 5 Geometric parameters of the studied shortened nozzles with tips

Fig. 6 shows the distribution of the Mach isoline for two types of nozzle tips with angles of 30^0 and 70^0 . The operation on the ground surface (P_H=1 bar) and at an inlet pressure of 100 bar is studied.

Just behind the corner point of the tip, the pressure increases sharply to a value close to atmospheric (P_H). For a smaller nozzle inlet angle (30^0) the transition to a constant pressure in the nozzle occurs more smoothly (in the area of ~ 20% of the nozzle length). But in general, changing the nozzle inlet angle does not greatly affect the flow structure and its characteristics. The initial pressure has a greater effect. With an increase in the pressure at the nozzle inlet (up to 100 bar), the pressure on the nozzle wall decreases. This fact is explained by the higher velocity of stream at the flow boundary outflowing of the conical nozzle part.

Behind the corner nozzle point (the transition of the cone into the tip) to its end between the boundary of the specified supersonic jet and the tip wall a developed separation zone with a large-scale vortex 2 is observed. Then the pressure in the flow is equalized with the ambient pressure. The free flow boundary 3 is determined by the

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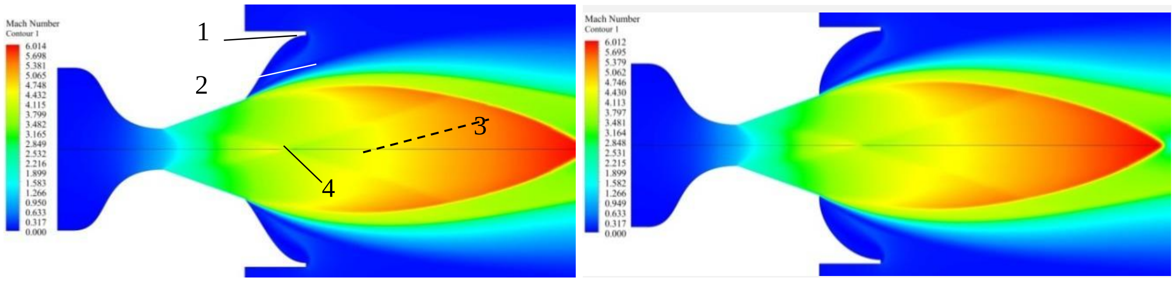
contour of the shortened conical nozzle part (the same for different shapes of the tip). Behind the critical cross section of the conical nozzle part in the flow a hanging saddle-shaped compaction wave 4 of low intensity is observed in front of the shock 1. A similar flow pattern is observed for other investigated nozzle configurations (500) under other equal conditions. Changing the inclination angle of the bell-shaped tip generatrix does not significantly change the value of the static pressure behind the corner point.



a) b)

Fig. 6 Mach number distribution in nozzles with a corner entrance to tip of 30^0 (a) and 70^0 (b) at P_0 = 100 bar, P_H = 1bar

EFFECT OF CHANGING EXTERNAL PRESSURE

Let us consider the effect of changing flight conditions on the wave structure of the nozzle flow. Figure 7 shows the Mach number distribution in a gas flow in the nozzle with a conical length section l = 5 mm at P_0 = 50 bar and an external pressure of P_H = 1 bar.

In the conical inlet section of the nozzle, the gas flow undergoes preliminary expansion, with the sonic line convex toward the nozzle outlet. Downstream of the nozzle conical inlet section the underexpanded flow with the characteristic wave structure is observed. A pendant shock wave 2, caused by a converging characteristic bundle, is observed in front of the free boundary 1 of the jet flowing from the nozzle. The shock is a result of the characteristic reflecting from the free boundary of the jet and occurring near the nozzle exit edge. A zone 5 of developed flow separation from the nozzle wall is observed beyond corner point 3 of the nozzle entrance 4. At the tip exit, a small secondary vortex zone 6 forms, caused by the large change gradient in the tip radius at its exit. In the gas jet core the flow accelerates unevenly; high velocities of up to Mach 5.5 are observed in the peripheral jet zone 8.

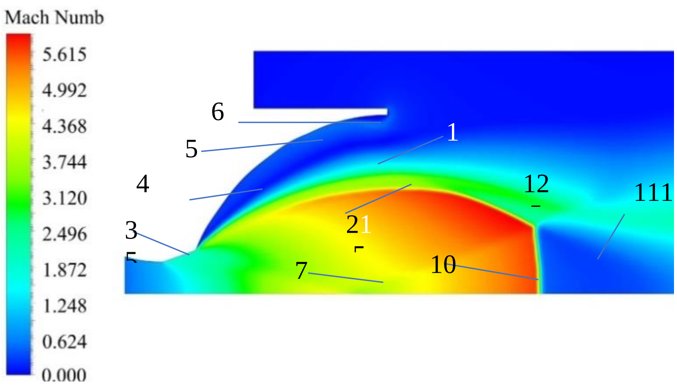


Fig. 7 Mach number distribution in a gas flow in nozzles with a conical section of length l = 5 mm at P0 = 50 bar and an external pressure of PH = 1 bar

Along the flow axis, in the direction of the tip exit, the flow velocity in zone 7 is lower than in zone 8. This is explained by the existence of a low-intensity, pendent "saddle-shaped" shock wave in the gas flow core downstream of the inlet conical nozzle section. Pendent shock wave 2 is caused by supersonic compression from a bundle of converging characteristics reflected from the free jet boundary. Pendent shock wave 2 closes with a Mach 10 disk, behind which the flow becomes subsonic in zone 11. The flow velocity is subsonic behind reflected

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shock wave 12, as well as behind the Mach disk 10 and the pressure is higher than the ambient one P_H. Therefore, behind both (the disk and the shock wave), the gas flow accelerates again, forming a second "barrel" of lower intensity. This is explained by wave energy losses in the flow in the first "barrel."

When flying at altitude, i.e., at an external pressure of P_H = 1 bar, the flow pattern changes (Fig. 8). Figure 8 shows the Mach number distribution in the gas flow at P_0 = 50 bar and P_H = 0.1 bar. On the nozzle wall, the flow is continuous; the hanging shock 1 originates from the corner point 2 of the nozzle entrance and extends to the nozzle exit 3, almost equidistant from the nozzle wall.

Behind shock wave 1, a low velocity zone 4 and high pressure forms in front of the nozzle wall, caused by the flow turning in the shock wave. Behind the tip exit, the flow from zone 4 with increased pressure exits the tip, forming an expanding flow 5. This flow distorts the shape of the hanging shock wave 1, in other words, it deforms the first "barrel" of the supersonic flow just behind the nozzle exit in zone 6. The free flow boundary 7 behind the exit deviates from the axis by an angle determined by the Prandtl-Meyer flow at the corner point of the nozzle exit.

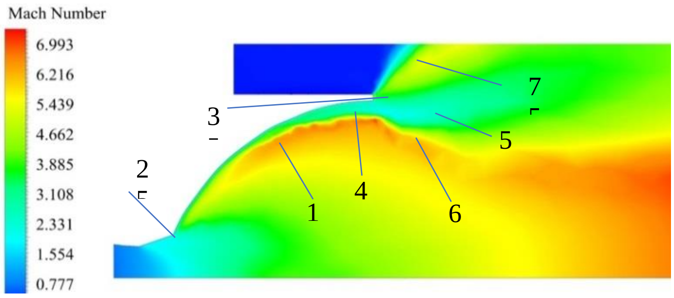

Fig. 8 Mach number distribution in a gas flow in nozzles with a conical section of length *l* = 5 mm at P_0 = 50 bar and an external pressure of P_H = 0.1 bar

The decrease in the intensity of hanging shock wave 1 as we approach corner point 2 of the nozzle inlet (upstream) is explained by a decrease in the radial displacement of the lines, which leads to a weakening of the flow overexpansion just behind corner point 2. The longitudinal dimensions of the first (distorted) "barrel" significantly exceed the barrel dimensions at P_H = 1.0 bar.

As the external pressure decreases from P_H = 0.1 to P_H = 0.01 bar, the configuration of the first "barrel" in the flow remains virtually unchanged. The flow at the free boundary changes proportionally to the decrease in external pressure. Free boundary 1 deflects in the Prandtl-Meyer flow by a correspondingly larger angle. The flow velocity in zone 2 also increases proportionally to P_H. It should be noted that the "saddle-shaped" wave structure in the first "barrel" of the flow at P_H = 0.1 bar and P_H = 0.01 bar is virtually absent. This is explained by the more uniform radial expansion of the flow within the first "barrel." Changing the initial pressure also affects the flow structure. The flow patterns in Fig. 3a) and Fig. 7 can be compared. The geometric parameters of the nozzles are the same, but the initial pressure is different: 50 bar (Fig. 7) and 100 bar (Fig. 3a).

THRUST CHARACTERISTICS

Formally, the thrust coefficient shows how much the supersonic nozzle part profiled along the streamline increases the nozzle thrust with a critical area at the outlet, and depends only on the flow expansion degree in the nozzle. For a shortened nozzle with a bell-shaped nozzle, Cf are determined taking into account the nozzle geometry and momentum losses in it. During the shortened nozzle design for dense layouts, a number of specific tasks arise that are not typical for the process of creating a round classical scheme nozzle (Laval). The main problem is determining the flow characteristics in the nozzle (especially momentum losses and stress distribution in the nozzle wall) and choosing the optimal parameters of the nozzle profile "compression".

Fig. 9 shows the calculated thrust coefficients (Cf) for experimental nozzles with total lengths *L* = 45, 40, and 35 mm and conical section lengths *l* = 5, 15, and 20 mm, as designated in Table 2.

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TABLE 1
GEOMETRIC PARAMETERS OF SHORTENED NOZZLES

N	**1**	**2**	**3**	**4**	**5**
l, mm	5	15	20	15	20
L, mm	35	35	35	40	45

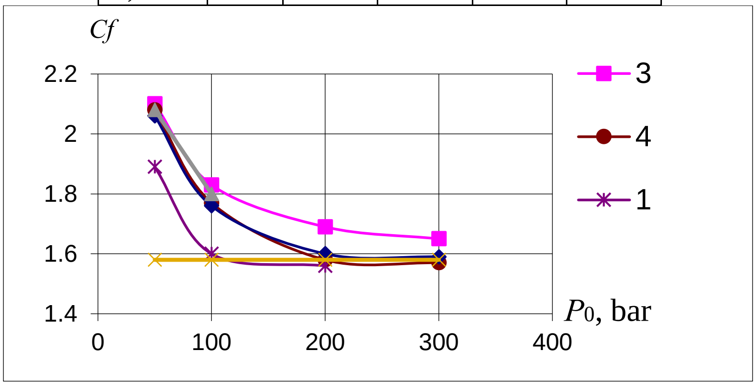

Fig. 9 Thrust coefficients of shortened nozzles with the parameters in Tab. 2 at PH = 1.0 bar (1-5) and PH = 0.1 bar (6)

A result analysis shows that changing the overall nozzle length, all other things being equal, does not significantly affect the thrust characteristics (see curves 2, 4-5).

However, changing the length of the conical nozzle section with the same overall nozzle length leads to a change in the thrust coefficient, although the nature of the dependence curves does not change (curves 1-3 in Fig. 9). In particular, in Fig. 9, curve 1 corresponds to the thrust coefficient of a shortened nozzle with the shortest conical section ($l = 5$ mm), curves 2 and 3 – to nozzles with a longer cone ($l = 15$ mm and 20 mm, respectively). Moreover, the thrust coefficient Cf of these nozzles increases proportionally to the conical section elongation.

With the same conical section length and an increase in the overall nozzle length, the thrust coefficient remains virtually unchanged (curves 2, 4). Thus, increasing the conical section length leads to an increase in the thrust coefficient, while with increasing inlet pressure, the thrust coefficient decreases and tends toward a constant value, obtained at low ambient pressure (i.e., flight outside the atmosphere). Curve 6 in Fig. 9 corresponds to these conditions. This pattern of dependence is explained by the changes in the wave and vortex structure of the flow in the nozzle, described above, in particular, the influence of the overexpansion degree of the gas flow in the tip at the relative (to the ambient pressure) pressure at the nozzle inlet changes, and the restructuring of the flow in the tip. An increase in the size of the developed separated flow zone in a nozzle with an elongated tip or a shortened conical nozzle section leads to the impulse value loss. This result coincides with the other calculation results of flow impulse characteristics in shortened nozzles using approximate methods and experimental studies of flows in shortened nozzles [Strelnikov, H.O., et al, (2025)].

REGRESSION ANALYSIS OF RESULTS

Using the ANSYS Fluent computational package, it is shown above that the flow patterns in the nozzle are affected by the length of the inlet conical part, the angle of entry into the nozzle, the total nozzle length and the operating conditions of the nozzle, i.e. the inlet pressure and the ambient pressure. It was found that the separation flow nature in the nozzle, and hence the thrust coefficient, depends on the expansion degree of the flow from the shortened Laval nozzle.

Using regression analysis, the main patterns of changes in the flow characteristics of shortened nozzle with a bell-shaped tip at changing the geometry and operating modes of the nozzle were identified. The analysis was carried out using the STATSGRAPHS-PLUS program.

Factor coding was carried out to convert the natural factor values into dimensionless quantities and the possibility of comparative influence assessment on the thrust coefficient of various parameters regardless of their dimension,

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The relationship between the coded and natural expression of the factor is given by the formula $x_i = \dfrac{X_i - X_{i0}}{\Delta X_i}$,

where Xi is the coded expression of the i-th factor; Xi is the natural factor value; Xi0 – value of the i-th factor at zero level; ΔXi – variation interval of the i-th factor. Table 2 shows the considered variation factor intervals.

TABLE 2
FACTOR CHANGE AREA

Factor	Factor code	Unit of measurement	Factor levels		
			-1	0	1
Nozzle cone length, l	X1	mm	5	10	20
Nozzle total length, L	X2	mm	30	35	45
Inlet pressure, P0	X3	bar	50	100	200
External pressure, PH	X4	bar	0,1	0,55	1,0

Second-order polynomial dependencies of the form are used:

$$Y = a_0 + \sum_{i}^{n} a_i \cdot x_i + \sum_{i}^{n} a_{ii} \cdot x_i^2 + \sum_{i<j}^{n} a_{ij} \cdot x_i \cdot x_j,$$

where Y is the response function (Cf); n = 4 is the number of variable factors.

Calculations showed that the dependence of Cf (l, L, P0, PH) has the form

$$Cf = 1{,}6 - 0{,}005L + 0{,}003 \cdot l + 0{,}003P_H + 0{,}001P_0 + 0{,}005l^2 - 0{,}003 \cdot l \cdot P_H + \\ + 0{,}002 \cdot l \cdot P_0 - 0{,}0068L \cdot P_0 - 0{,}008P_H \cdot P_0 + 0{,}003LP_H \quad (1)$$

The adequacy of the obtained model is confirmed by the high value of the determination coefficient R2 = 93.4%. The most convenient and visual significance of the model coefficients can be checked using the Pareto Chart. Based on the Pareto chart, we can identify significant coefficients in formula (1), i.e., those whose values extend beyond the vertical line. These coefficients enter the relationship expression with different signs, as reflected in Fig. 10 by the different colors of the bars. Thus, we obtain the final form of the relationship.

$$Cf = 1{,}6 - 0{,}005L + 0{,}003l + 0{,}005l^2 - 0{,}0068L \cdot P_0 - 0{,}008P_H \cdot P_0 \quad (2)$$

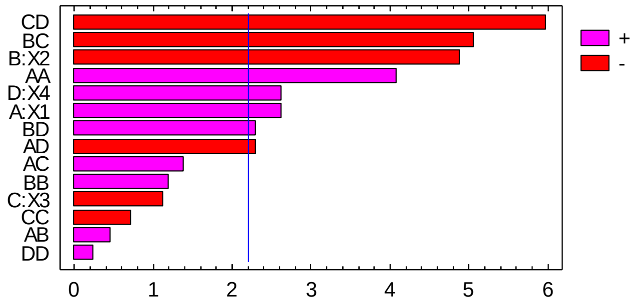

Fig. 10 Pareto Chart

The dependence of the thrust coefficient on the selected parameters can be clarified by three-dimensional graphs of the dependence of the Cf on a pair of parameters. Fig. 11 a) shows the graph of the dependence of the function Cf(*l*, *L*) on the factors *l* and *L* (the trust coefficient on the geometric parameters and P_0, P_H (trust coefficient from the pressure). In this case, the factors P_0 and P_H are fixed at the middle level (that is, X3, X4 at the zero level).

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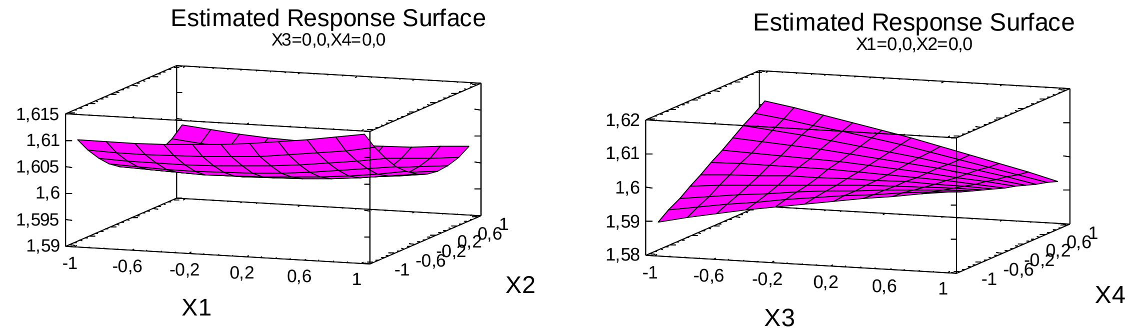

Fig. 11 - Dependence of the function Cf(*l*, *L*) (a), Cf(P_0, P_H)

It can be seen that, on one hand, an increase in the conical part length leads to a slight increase in the thrust coefficient; the dependence is linear (Fig. 11 a). On other hands, the dependence of Cf on the total nozzle length is nonlinear. The lowest Cf value at fixed X3 = X4 = 0, i.e. P_0 = 0.5 bar and P_H = 100 bar, is observed at the average value of the total nozzle length (L = 35 mm). At decrease or increase in the nozzle length, the coefficient increases.

The dependence of the thrust coefficient on the inlet pressure and the ambient pressure downstream of the tip Cf(P_0, P_H), is different one (Fig. 11 b). A direct dependence between the coefficient and the inlet pressure is observed: Cf increases with increasing pressure. However, there is an inverse dependence with the ambient pressure: as the ambient pressure increases, Cf decreases. It means that under terrestrial conditions, thrust coefficient losses increase compared to operation at altitude.

CONCLUSIONS

The flow characteristics in a shortened circular nozzle with a bell-shaped tip are studied. At low nozzle inlet pressures and high external pressure, a developed separation zone is observed between the jet boundary and the tip wall, extending upstream from the tip exit to the shortened nozzle end. The characteristics of this zone depend on the flow expansion ratio in the shortened nozzle.

At high flow underexpansion, the jet boundary in the tip converges to the nozzle wall, distorting the first "barrel" shape of the jet. At high external pressure, disturbances from the external environment are transmitted along the boundary layer of the jet adjoining to the tip wall.

A significant influence of the relative length of the shortened conical nozzle section preceding the tip on the flow characteristics in the nozzle was found. At the same time overall nozzle length (with the same geometric expansion ratio) and the nozzle entry angle have less influence. The nozzle thrust coefficient decreases with increasing relative pressure at the nozzle inlet due to the reduced influence of external pressure on the flow in the tip, reaching a constant value after the flow converge to the tip wall beyond the corner transition point. At higher flow miscalculation degrees, the thrust coefficient is higher for a nozzle with a longer conical section.

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